

The University of Zambia
School of Natural Sciences
Department of Mathematics & Statistics
2013/14 ACADEMIC YEAR FINAL EXAMINATIONS
MAT1100 – FOUNDATION MATHEMATICS
14th July, 2014

- INSTRUCTIONS:**
- (1) Write down your **Computer number** and the **TG number** on each answer booklet used.
 - (2) Answer any **Five (5)** questions only.
 - (3) Show all essential working to avoid loss of marks.
 - (4) Write down the **question numbers** of all the questions attempted in the first column on the cover of the main answer booklet.
 - (5) Calculators and tables are **not allowed** in this paper

TIME ALLOWED: Three (3) hours.

Q1. (a) The sets A, B, C all intersect and U is the universal set. Shade the part described by the set $[B \cup (A' \cap C')]'$ in a Venn diagram.

(b) The functions f and g are defined by

$$f(x) = \frac{4}{x-2}, \quad x \in \mathbb{R}, x \neq 2$$

$$g(x) = x + 2, \quad x \in \mathbb{R}.$$

Find

(i) the domain of the function $g \circ f$

(ii) $(g \circ f)^{-1}(x)$.

(c) Use Cramer's rule to solve the system of equations:

$$\begin{array}{rcl} x + 2y + 3z & = & -2 \\ -2x & + & z = 3 \\ x - 3y & = & 5 \end{array}$$

(d) Find the equation of the tangent to the curve

$$y = x^2 - 9x^{-1},$$

at the point $(3, 6)$.

Q2. (a) Find the solution set of each of the following inequalities:

(i) $|3x - 2| > 4$

(ii) $\frac{2}{x-3} < \frac{3}{2x-1}$

(b) Find the equation of a circle which passes through the points (1,4), (7,5) and (1,8).

(c) (i) Find the period and phase shift for the function

$$f(x) = 1 - 3\sin(2x + \pi),$$

and hence or otherwise,

(ii) sketch the curve for $-\pi \leq x \leq \pi$.

(d) Differentiate the function $f(x) = \frac{2}{\sqrt{3x-1}}$ from first principle.

Q3. (a) Evaluate the limits:

(i) $\lim_{x \rightarrow 3} \frac{x^3 - 27}{x^2 - 9}$ (ii) $\lim_{x \rightarrow +\infty} \frac{2x^2 + 1}{6 + x - 3x^2}$

(b) The line $y = 5x - 13$ meets the circle $(x - 2)^2 + (y + 3)^2 = 26$ at the points A and B .

(i) Show that A has coordinates (1, -8) and B has coordinates (3, 2).

M is the midpoint of the line AB .

(ii) Find the equation of the line which passes through M and is perpendicular to the line AB . Write your answer in the form $ax + by + c = 0$, where a , b and c are integers.

(c) Express each of the following in the form $r(\cos\theta + i\sin\theta)$:

(i) $(3 - 3i\sqrt{3})(1 - i)$

(ii) $\frac{-2 - i\sqrt{3}}{-2 + i\sqrt{3}}$

(d) Show that $x = 0$ is a solution of the equation

$$\begin{vmatrix} x-1 & 4 & -1 \\ 1 & x+2 & 1 \\ 2x-4 & 4 & x-4 \end{vmatrix} = 0,$$

and find the other two roots.

- Q4. (a) Given the sets $A = [-7, 3]$, $B = [-3, 3]$, $C = (-1, 6)$ and let $\mathbb{R}$ be the universal set. Find each of the following sets and represent it on a number line:

(i) $A - B$

(ii) $A \cup (B \cap C)'$

- (b) Evaluate each of the following integrals:

(i) $\int \frac{3x}{\sqrt{1-2x^2}} dx$

(ii) $\int \frac{x^3 + 3}{x^2 - 1} dx$

(iii) $\int_0^{\pi/2} x \cos x dx$

- (c) (i) Given that $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 3 & 6 \\ 1 & 2 & 3 \end{pmatrix}$, find A^{-1} .

(ii) Hence or otherwise, find X given that $Y = AX$ and $Y = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$.

- (d) Solve the equation

$$|x^2 - 3x| = -4x + 6$$

- Q5. (a) For each of the following, find $\frac{dy}{dx}$:

(i) $y = e^{4x}(3x^2 - 5)^6$

(ii) $x^2 + \sin y - y^2 = 7$

- (b) The roots of the equation

$$9x^2 + 6x + 1 = 4kx,$$

where k is a real constant, are denoted by α and β .

- (i) Show that the equation whose roots are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ is

$$x^2 + 6x + 9 = 4kx.$$

- (ii) Find the set of values of k for which α and β are real.

- (c) Solve each of the following trigonometric equations for x , if $0 \leq x \leq 2\pi$:

(i) $\cos^2 x - \sin x - 1 = 0$

(ii) $\cos 2x + 3\sin x - 2 = 0$

- (d) Use the principle of mathematical induction to prove that for $n \in \mathbb{Z}^+$,

$$8^n - 1 \text{ is divisible by } 7.$$

Q6. (a) (i) Express $(1-i)^6$ in the form $p+iq$, where p and q are integers.

(ii) Expand $\frac{3}{1+3x}$ as a series in ascending powers of x , up to and including the term in x^3 , indicating the range of values of x for which the expansion is valid.

(b) Solve the equation for x :

$$3^{2x+1} + 5 = 16(3^x).$$

(c) A cylindrical can, with no lid, has a circular base of radius r cm. The total surface area of the can is $300\pi \text{ cm}^2$.

(i) Show that the volume $V \text{ cm}^3$ of the can is given by

$$V = \frac{\pi}{2} r(300 - r^2).$$

(ii) Find the value of r for which V is maximum.

(d) Prove each of the following identities:

(i) $\cot \theta + \tan \theta \equiv \sec \theta \csc \theta$

(ii) $\frac{\cos A + \tan A}{\sin A \cos A} \equiv \csc A + \sec^2 A$

Q7. (a) Given that $(3x+2)$ is a factor of $f(x) = 3x^3 + Ax^2 - 4x - 4$.

(i) Show that $A = 5$.

(ii) When $A = 5$, factorize $f(x)$ completely.

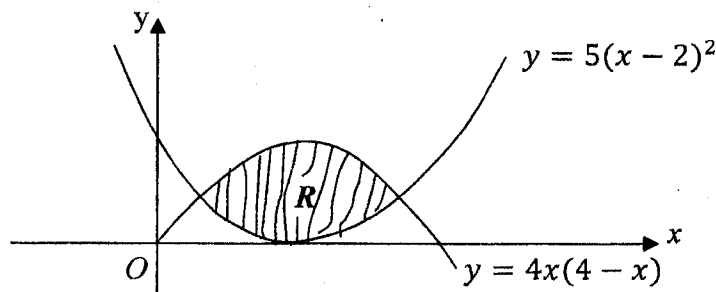
(iii) Hence, sketch the graph of $y = f(x)$, indicating the x - and y -intercepts and the turning points.

(b) Determine whether the function $f(x) = 2x - 5x^3$ is even or odd or neither.

(c) Solve the equation

$$z^3 + 32\sqrt{3} + 32i = 0$$

(d) The diagram shows the shaded region R which is bounded by the curves $y = 4x(4-x)$ and $y = 5(x-2)^2$. Find the area of the shaded region R .



END OF EXAMINATION